## VASAVI COLLEGE OF ENGINEERING (AUTONOMOUS), HYDERABAD

Accredited by NAAC with A++ Grade
B.E. III-Semester Main \& Backlog Examinations, Jan./Feb.-2024

## Discrete Mathematics

(I.T.)

Time: $\mathbf{3}$ hours

Note: Answer all questions from Part-A and any FIVE from Part-B
Part-A $(10 \times 2=20$ Marks $)$

| Q. No. | Stem of the question | M | L | CO | PO |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | Explain Bi-implication with an example. | 2 | 1 | 1 | 1,2,12 |
| 2. | Distinguish among contradiction and contingency. | 2 | 1 | 1 | 1,2,12 |
| 3. | Prove that if $2^{p}-1$ is prime then ' $p$ ' is also prime. | 2 | 1 | 2 | 1,2,12 |
| 4. | Find the total number of positive divisors $\tau(n)$ and sum of the positive divisors $\sigma(\mathrm{n})$ for the number $\mathrm{n}=14553$. | 2 | 2 | 2 | 1,2,12 |
| 5. | Explain Generalized Pigeonhole principle with an example. | 2 | 1 | 3 | 1,2,12 |
| 6. | Solve the recurrence relationa $a_{n}-3 a_{n-1}=5.7^{n}$ for $n \geq 1$ given that $\mathrm{a}_{0}=2$. | 2 | 2 | 3 | 1,2,12 |
| 7. | If $A=\{1,2,3,4\}$ then write an example of a relation on $A$ which is <br> (i)Reflexive, Symmetric but not Transitive. <br> (ii) Symmetric, Transitive but not reflexive. | 2 | 1 | 4 | 1,2,12 |
| 8. | Explain the procedure for constructing Hasse diagram. | 2 | 1 | 4 | 1,2,12 |
| 9. | If $\mathrm{G}=(\mathrm{V}, \mathrm{E})$ is a simple graph then Prove that $2\|E\| \leq\|V\|^{2}-\|V\|$. | 2 | 2 | 5 | 1,2,12 |
| 10. | Define planar graph with an example and Verify the Euler's formula for planar graphs with that example. $\text { Part-B }(5 \times 8=40 \text { Marks })$ | 2 | 1 | 5 | 1,2,12 |
| 11. a) | Verify whether $(\mathrm{p} \rightarrow \mathrm{q}) \rightarrow \mathrm{r}$ and $\mathrm{p} \rightarrow(\mathrm{q} \rightarrow \mathrm{r})$ are equivalent. | 4 | 3 | 1 | 1,2,12 |
| b) | Use quantifiers to express the following: <br> a) Every computer science student needs a course in discrete mathematics. <br> b) There is a student in this class who owns a personal computer. <br> c) Every student in this class has taken atleast one computer science course. <br> d) Every student in this class has been in atleast one room of every building on campus. | 4 | 1 | 1 | 1,2,12 |
| 12. a) | Prove that <br> (i) The Sum of even integer and odd integer is odd. <br> (ii) Prove that the square of any odd integer is of the form $8 \mathrm{k}+1$, for some integer ' $k$ '. | 4 | 2 | 2 | 1,2,12 |

b) Apply solution criteria to solve the Linear congruence
13. a) Find the no. of Permutations of the letters of the word "MISSISSIPPI". How many of these begin with 'I' ? Also how many of these begin with ' S ' and end with ' S '.
b) Solve the recurrence relation of the Fibonacci series of numbers.
14. a) Prove that congruence relation is a Equivalence relation.
b) Let $\mathrm{A}=\{1,2,3,4,6,8,12\}$, define a relation ' R ' on A such that $x R y$ iff $x$ divides $y$, then (i)Prove that $(\mathrm{A}, \mathrm{R})$ is a Poset (ii) Draw the Hasse diagram of $R$ (iii) Find the Maximal, Minimal, Greatest \& Least elements if any.
15. a) Define (i) Simple Graph (ii) Complete Graph (iii) Bipartite Graph (iv) Complete Bipartite Graph with one example each.
b) Let $\mathrm{G}=(\mathrm{V}, \mathrm{E})$ be a simple graph of order ' n ' and size ' m '. If G is bipartite graph then prove that $4 m \leq n^{2}$.
16. a) Prove that the generalization of Demorgan's law $\overline{\cap_{l=1}^{n} A_{l}}=\bigcup_{i=1}^{n} \overline{A_{i}}$, $\forall$ positive integer $n>1$, using Mathematical Induction.
b) Define Euler's $\varphi$ - function and also find the number of positive integers which are less than 25200 that are relatively prime to 25200 .
17. Answer any two of the following:
a) State and Prove Pascal's Identity.
b) Let $\mathrm{A}=\{1,2,3,4\}, \mathrm{B}=\{\mathrm{w}, \mathrm{x}, \mathrm{y}, \mathrm{z}\}$ and $\mathrm{C}=\{5,6,7\}$. Also let $\mathrm{R}_{1}$ be a relation from $A$ to $B ; R_{2}$ and $R_{3}$ be the relations from $B$ to $C$ defined by $\mathrm{R}_{\mathrm{l}}=\{(1, \mathrm{x})(2, \mathrm{x})(3, \mathrm{y})(3, \mathrm{z})\}$;
$\mathrm{R}_{2}=\{(\mathrm{w}, 5)(\mathrm{x}, 6)\} ; \mathrm{R}_{3}=\{(\mathrm{w}, 5)(\mathrm{w}, 6)\}$ then find (i) $R_{1} o R_{2}$ and $R_{1} o R_{3}$ (ii) $M\left(R_{1}\right) \quad, \quad M\left(R_{2}\right) \quad \& \quad M\left(R_{1} O \quad R_{2}\right)$ (iii) Verify that $M\left(R_{1} \circ R_{2}\right)=M\left(R_{1}\right) \cdot M\left(R_{2}\right)$.
c) In every graph Show that the number of vertices of odd degree is even.

| 4 | 3 | 2 | $1,2,12$ |
| :---: | :---: | :---: | :---: |
| 4 | 2 | 3 | $1,2,12$ |
|  |  | - |  |
| 4 | 3 | 3 | $1,2,12$ |
| 4 | 2 | 4 | $1,2,12$ |
| 4 | 3 | 4 | $1,2,12$ |$|$


| 4 | 1 | 5 | $1,2,12$ |
| :--- | :--- | :--- | :--- |
| 4 | 2 | 5 | $1,2,12$ |

$4311,2,12$

| 4 | 2 | 2 | $1,2,12$ |
| :---: | :---: | :---: | :---: |
|  |  | - |  |
| 4 | 2 | 3 | $1,2,12$ |
| 4 | 3 | 4 | $1,2,12$ |

$\begin{array}{lll}4 & 2 & 5\end{array} 1,2,12$

M : Marks; L: Bloom's Taxonomy Level; CO; Course Outcome; PO: Programme Outcome

| i) | Blooms Taxonomy Level - 1 | $27.50 \%$ |
| :---: | :--- | :---: |
| ii) | Blooms Taxonomy Level -2 | $40 \%$ |
| iii) | Blooms Taxonomy Level $-3 \& 4$ | $32.5 \%$ |

